

-1-

## LYGB - MPI PAPER 0 - QUESTION 1

$$x^2 + 5px + 2p = 0, \quad p \text{ CONSTANT}$$

"HAS REAL ROOTS"  $\Rightarrow$  2 DISTINCT REAL ROOTS ( $b^2 - 4ac > 0$ )  
 $\Rightarrow$  1 REPEATED REAL ROOT ( $b^2 - 4ac = 0$ )

THUS WE HAVE

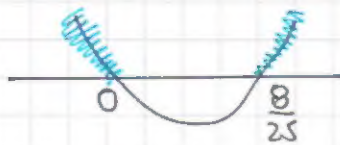
$$b^2 - 4ac \geq 0$$

$$(5p)^2 - 4 \times 1 \times 2p \geq 0$$

$$25p^2 - 8p \geq 0$$

$$p(25p - 8) \geq 0$$

CRITICAL VALUES ARE  $\begin{matrix} 0 \\ \frac{8}{25} \end{matrix}$



$$\therefore \underline{p \leq 0 \quad \text{or} \quad p \geq \frac{8}{25}}$$

# IYGB - MPI PAPER 0 - QUESTION 2

## GRADIENT OF L

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{-2 - 2} = \frac{-2}{-4} = \frac{1}{2}$$

## EQUATION OF L, GRADIENT $\frac{1}{2}$ , PASSING THROUGH (2, 5)

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - 5 = \frac{1}{2}(x - 2)$$

$$\Rightarrow 2y - 10 = x - 2$$

$$\Rightarrow 2y = x + 8$$

## WITH $x = 0$

$$2y = 8$$

$$y = 4$$

$$\therefore P(0, 4)$$

## WITH $y = 0$

$$0 = x + 8$$

$$x = -8$$

$$\therefore Q(-8, 0)$$

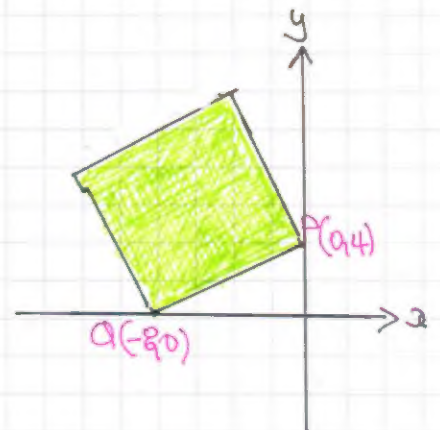
## LENGTH OF PQ IS GIVEN BY

$$\Rightarrow d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$\Rightarrow |PQ| = \sqrt{(0 - 4)^2 + (-8 - 0)^2}$$

$$\Rightarrow |PQ| = \sqrt{16 + 64}$$

$$\Rightarrow |PQ| = \sqrt{80}$$



## AREA OF SQUARE IS

$$\sqrt{80} \times \sqrt{80} = \underline{80}$$



# 1YGB - MPI PAGE 0 - QUESTION 3

a)  $f(x) = 4x^3 - 8x^2 - x + k$

$$\begin{aligned} \underline{x-2 \text{ IS A FACTOR}} &\Rightarrow f(2) = 0 \\ &\Rightarrow 4 \times 2^3 - 8 \times 2^2 - 2 + k \\ &\Rightarrow 32 - 32 - 2 + k \\ &\Rightarrow k = 2 \end{aligned}$$

$$\underline{f(x) = 4x^3 - 8x^2 - x + 2}$$

$$\begin{aligned} f\left(\frac{1}{2}\right) &= 4\left(\frac{1}{2}\right)^3 - 8\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 2 \\ &= \frac{1}{2} - 2 - \frac{1}{2} + 2 \\ &= 0 \end{aligned}$$

$\therefore \underline{(2x-1)}$  IS INDEED ALSO A FACTOR //

c) BY INSPECTION WE HAVE

$$f(x) = 4x^3 - 8x^2 - x + 2 = (x-2)(2x-1)(2x+1) //$$

d)  $4\sin^3 y - 8\sin^2 y - \sin y + k = 0$   
 $\Rightarrow (\sin y - 2)(2\sin y - 1)(2\sin y + 1) = 0$   
 (PART a)

$$\Rightarrow \sin y = \begin{cases} \cancel{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{cases}$$

# 1YGB - MPI PAGE 0 - QUESTION 3

## SOLVING SEPARATELY

$$\bullet \arcsin\left(\frac{1}{2}\right) = 30^\circ$$


$$\begin{cases} y = 30^\circ \pm 360^\circ n \\ y = 150^\circ \pm 360^\circ n \end{cases}$$

$$n = 0, 1, 2, 3, \dots$$

$$\bullet \arcsin\left(-\frac{1}{2}\right) = -30^\circ$$

$$\begin{cases} y = -30^\circ \pm 360^\circ n \\ y = 210^\circ \pm 360^\circ n \end{cases}$$

$$n = 0, 1, 2, 3, \dots$$


$$\underline{y = 30^\circ, 150^\circ, 210^\circ, 330^\circ}$$

# IYGB - MPI PAPER 0 - QUESTION 4

a) FINDING THE AREA BY INTEGRATION

$$\text{AREA} = \int_{x_1}^{x_2} f(x) dx = \int_1^4 x^2 - 2x + 2 dx$$

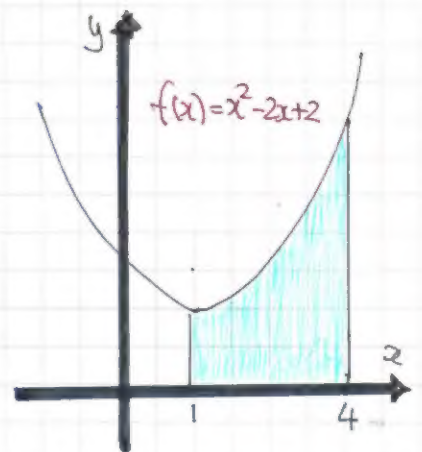
$$= \left[ \frac{1}{3}x^3 - x^2 + 2x \right]_1^4$$

$$= \left( \frac{64}{3} - 16 + 8 \right) - \left( \frac{1}{3} - 1 + 2 \right)$$

$$= \frac{40}{3} - \frac{4}{3}$$

$$= \frac{36}{3}$$

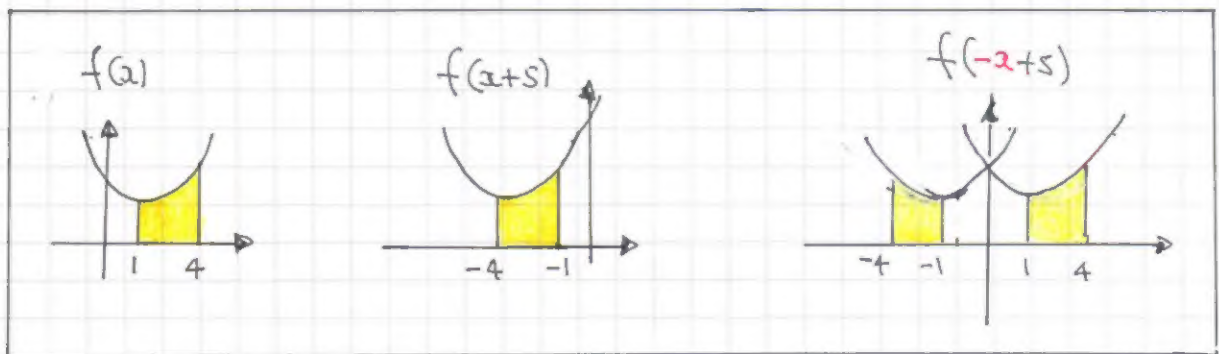
$$= 12 //$$



b) NOW WE HAVE USING THE PROPERTIES OF THE INTEGRAL & TRANSFORMATION

$$\int_1^4 2f(5-x) dx = 2 \int_1^4 f(5-x) dx = 2 \int_1^4 f(x) dx = 2 \times 12 = 24 //$$

(See Below)





## IYGB - MPI PAPER 0 - QUESTION 5

a)

$$\underline{a} = 3\underline{i} - 2\underline{j}$$

$$\underline{b} = 5\underline{i} + 4\underline{j}$$

$$AB : BC = 2 : 5$$

USING POSITION VECTORS

$$\Rightarrow \vec{OC} = \vec{OB} + \vec{BC}$$

$$\Rightarrow \vec{OC} = \vec{OB} + \frac{5}{2} \vec{AB}$$

$$\Rightarrow \underline{c} = \underline{b} + \frac{5}{2} (\underline{b} - \underline{a})$$

$$\Rightarrow \underline{c} = \underline{b} + \frac{5}{2} \underline{b} - \frac{5}{2} \underline{a}$$

$$\Rightarrow \underline{c} = \frac{7}{2} \underline{b} - \frac{5}{2} \underline{a}$$

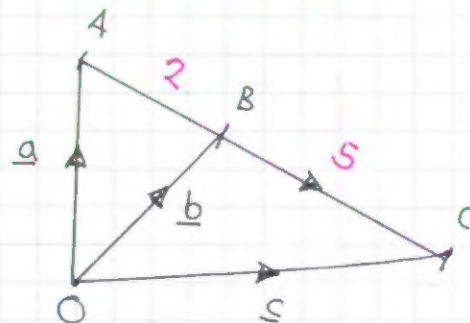
$$\Rightarrow \underline{c} = \frac{1}{2} (7\underline{b} - 5\underline{a})$$

$$\Rightarrow \underline{c} = \frac{1}{2} [7(5\underline{i} + 4\underline{j}) - 5(3\underline{i} - 2\underline{j})]$$

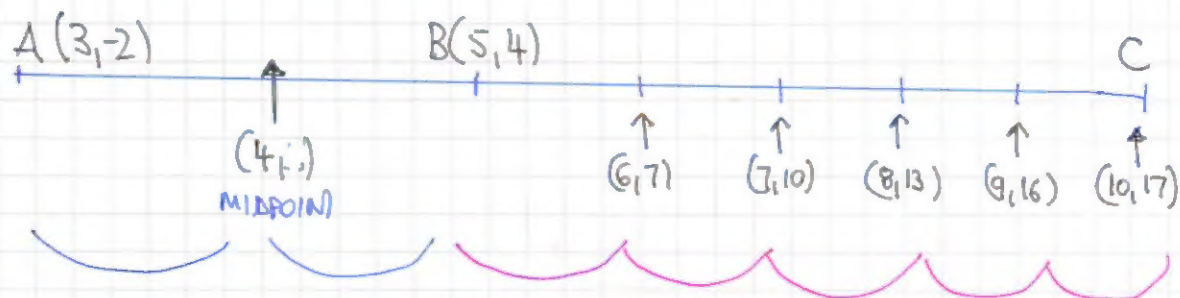
$$\Rightarrow \underline{c} = \frac{1}{2} [35\underline{i} + 28\underline{j} - 15\underline{i} + 10\underline{j}]$$

$$\Rightarrow \underline{c} = \frac{1}{2} [20\underline{i} + 38\underline{j}]$$

$$\Rightarrow \underline{c} = \underline{10\underline{i} + 19\underline{j}}$$



OR SIMPLY BY INSPECTION



$$\therefore \underline{c} = \underline{10\underline{i} + 19\underline{j}}$$

AS BEFORE

## 1YGB - MPI PAPER 0 - QUESTION 5

- b)
- $\vec{AB} = \underline{b} - \underline{a} = (5\underline{i} + 4\underline{j}) - (3\underline{i} - 2\underline{j}) = 2\underline{i} + 6\underline{j}$
  - DIRECTION CAN BE SCALED TO  $\underline{i} + 3\underline{j}$
  - Hence since  $|\underline{i} + 3\underline{j}| = \sqrt{1^2 + 3^2} = \sqrt{10}$ ,  
we need 6 "vector steps" in further direction from B
  - i.e.  $\underline{d} = \underline{b} + 6(\underline{i} + 3\underline{j}) = 5\underline{i} + 4\underline{j} + 6\underline{i} + 18\underline{j}$   
 $\underline{d} = \underline{b} - 6(\underline{i} + 3\underline{j}) = 5\underline{i} + 4\underline{j} - 6\underline{i} - 18\underline{j}$   
 $\therefore \underline{d} = 11\underline{i} + 22\underline{j} \quad \text{or} \quad \underline{d} = -\underline{i} - 14\underline{j}$

### ALTERNATIVE

- Let  $D(a, b)$
- GRADIENT  $AB = \frac{4 - (-2)}{5 - 3} = \frac{6}{2} = 3$
- LINE THROUGH  $A, B$  &  $D$  IS  

$$\begin{aligned} y - 4 &= 3(x - 5) \\ y - 4 &= 3x - 15 \\ y &= 3x - 11 \end{aligned}$$
- Hence  $D(a, 3a - 11)$
- NOW THE DISTANCE  $|BD| = 6\sqrt{10}$   
 $\Rightarrow \sqrt{(3a - 11 - 4)^2 + (a - 5)^2} = 6\sqrt{10}$   
 $\Rightarrow (3a - 15)^2 + (a - 5)^2 = 360$

- 3 -

1YGB - MPI PAPER 0 - question 5

$$\Rightarrow \begin{cases} 9a^2 - 90a + 225 \\ a^2 - 10a + 25 \end{cases} = 360$$

$$\Rightarrow 10a^2 - 100a + 250 = 360$$

$$\Rightarrow a^2 - 10a + 25 = 36$$

$$\Rightarrow a^2 - 10a - 11 = 0$$

$$\Rightarrow (a+1)(a-11) = 0$$

$$\Rightarrow a = \begin{cases} -1 \\ 11 \end{cases} \quad b = \begin{cases} 3(-1) - 11 = -14 \\ 3(11) - 11 = 22 \end{cases}$$

$$\therefore D(-1, -14) \quad \text{or} \quad (11, 22)$$

$$\therefore \underline{d = -1 - 14} \quad \text{or} \quad \underline{d = 11 + 22}$$



# IYGB - MPI PAPER 0 - QUESTION 6

THE DERIVATIVE IS FORMALLY GIVEN BY

$$f'(x) = \lim_{h \rightarrow 0} \left[ \frac{f(x+h) - f(x)}{h} \right]$$

IN THIS CASE WE HAVE

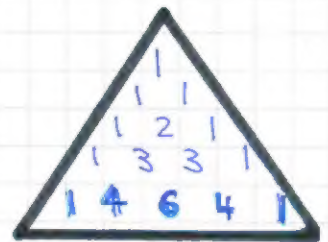
$$f(x) = x^4$$

$$f(x+h) = (x+h)^4$$

EXPANDING BINOMIALLY WE HAVE

$$(x+h)^4 = 1x^4h^0 + 4x^3h^1 + 6x^2h^2 + 4x^1h^3 + 1x^0h^4$$

$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$



TIDYING UP NEXT

$$f(x+h) - f(x) = (x+h)^4 - x^4 = (\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - \cancel{x^4}$$

BNARY WE HAVE

$$f'(x) = \lim_{h \rightarrow 0} \left[ \frac{f(x+h) - f(x)}{h} \right] = \lim_{h \rightarrow 0} \left[ \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \cancel{4x^3} + \cancel{6x^2h} + \cancel{4xh^2} + \cancel{h^3} \right]$$

$$= \underline{4x^3}$$

# IYGB - MPI PAPER 0 - QUESTION 7

$$m = 20e^{0.02t}, t \geq 0$$

$m$  = MASS IN kg

$t$  = TIME IN HOURS

a) FIRSTLY WITHIN  $t=0$

$$m = 20 \times e^0$$

$$m = 20 \text{ kg.} \leftarrow \text{INITIAL MASS}$$

"THREE TIMES THE INITIAL MASS" ....

$$60 = 20e^{0.02t}$$

$$3 = e^{0.02t}$$

$$\ln 3 = 0.02t$$

$$\ln 3 = \frac{1}{50}t$$

$$t = 50 \ln 3 \approx 54.93$$

$\therefore$  APPROX 55 HOURS

b) RATE OF CHANGE  $\Rightarrow$  DIFFERENTIATION

$$\bullet m = 20e^{0.02t}$$

$$\frac{dm}{dt} = 20 \times 0.02 \times e^{0.02t}$$

$$\frac{dm}{dt} = 0.4 \times e^{0.02t}$$

$$\bullet m = 100$$

$$100 = 20e^{0.02t}$$

$$5 = e^{0.02t}$$

COMBINING WE OBTAIN

$$\left. \frac{dm}{dt} \right|_{m=100} = 0.4 \times e^{0.02t} = 0.4 \times 5 = \underline{\underline{2 \text{ kg h}^{-1}}}$$

# YGB - MPI PAGE 0 - QUESTION 8

REWRITE THE EQUATION IN INDICIAL FORM & DIFFERENTIATE

$$\Rightarrow y = x(x^2 - 128\sqrt{x})$$

$$\Rightarrow y = x(x^2 - 128x^{\frac{1}{2}})$$

$$\Rightarrow y = x^3 - 128x^{\frac{3}{2}}$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 192x^{\frac{1}{2}}$$

FOR STATIONARY POINTS SET  $\frac{dy}{dx} = 0$

$$\Rightarrow 3x^2 - 192x^{\frac{1}{2}} = 0$$

$$\Rightarrow x^2 - 64x^{\frac{1}{2}} = 0$$

$$\Rightarrow x^2 = 64x^{\frac{1}{2}}$$

$$\Rightarrow \frac{x^2}{x^{\frac{1}{2}}} = 64 \quad (\text{WE ARE NOT CONCERNED WITH } x=0)$$

$$\Rightarrow x^{\frac{3}{2}} = 64$$

$$\Rightarrow \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = (64)^{\frac{2}{3}}$$

$$\Rightarrow x^1 = (\sqrt[3]{64})^2$$

$$\Rightarrow x = 16$$

CHECK THE NATURE OF THE POINT BY THE SECOND DERIVATIVE TEST

$$\Rightarrow \frac{d^2y}{dx^2} = 6x - 96x^{-\frac{1}{2}}$$

$$\Rightarrow \left. \frac{d^2y}{dx^2} \right|_{x=16} = 6 \times 16 - 96 \times 16^{-\frac{1}{2}} = 96 - 96 \times \frac{1}{4} = 96 - 24 = 72 > 0$$

LOCAL MINIMUM



1YGB - MPI PAPER 0 - QUESTION 8

FINALLY TO FIND THE  $y$  CO-ORDINATE IN THE REQUIRED FORM

$$y = x(x^2 - 128\sqrt{x})$$

$$y = 16(16^2 - 128\sqrt{16})$$

$$y = -4096$$

$$y = -2^{12} \quad (\text{TRIAL \& ERROR OF POWER OF 2})$$

$\therefore$  LOCAL MINIMUM AT  $(16, -2^{12})$



# 1YGB - MPI PAPER 0 - QUESTION 9

a)

START BY OBTAINING THE "PARTICULARS" OF THE TWO CIRCLES

$$\bullet x^2 + y^2 - 6x - 2y = 15$$

$$x^2 - 6x + y^2 - 2y = 15$$

$$(x-3)^2 - 9 + (y-1)^2 - 1 = 15$$

$$(x-3)^2 + (y-1)^2 = 25$$

CENTRE AT (3,1)

RADIUS 5

$$\bullet x^2 + y^2 - 18x + 14y = 95$$

$$x^2 - 18x + y^2 + 14y = 95$$

$$(x-9)^2 - 81 + (y+7)^2 - 49 = 95$$

$$(x-9)^2 + (y+7)^2 = 225$$

CENTRE AT (9,-7)

RADIUS 15

IF THE DISTANCE BETWEEN THEIR CENTRES IS ....

•  $15 + 5 = 20$  , THEY ARE TOUCHING EXTERNALLY

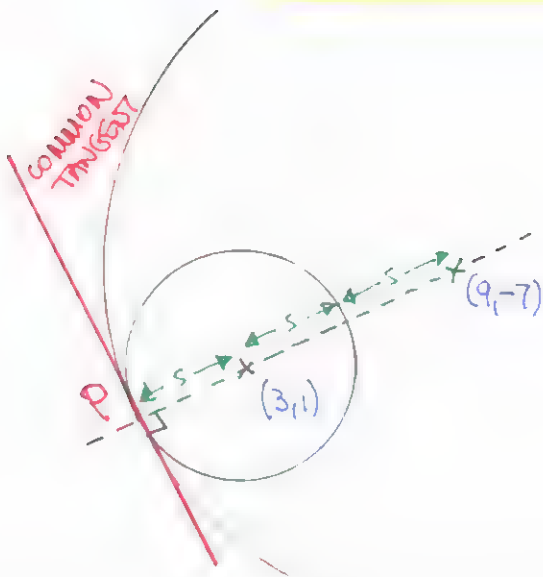
•  $15 - 5 = 10$  , THEY ARE TOUCHING INTERNALLY

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$d = \sqrt{(-7-1)^2 + (9-3)^2}$$

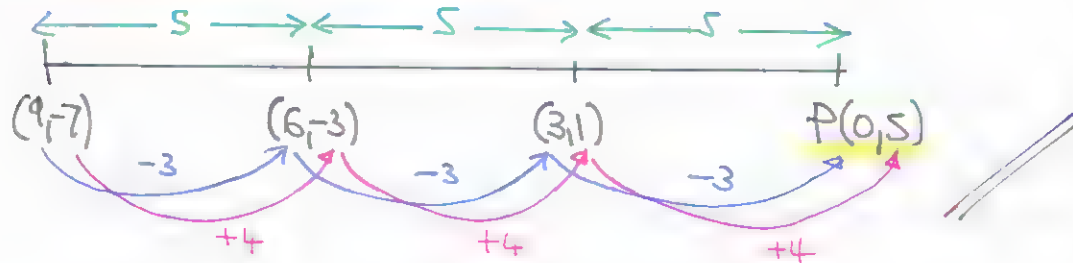
$$d = \sqrt{64 + 36} = 10$$

INDEED THEY ARE TOUCHING INTERNALLY



# LYGB - MPI PAPER 0 - QUESTION 9

b) BY INSPECTION WE HAVE



c) GRADIENT OF COMMON RADIUS, USING (9, -7) & (3, 1)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-7)}{3 - 9} = \frac{8}{-6} = -\frac{4}{3}$$

GRADIENT OF THE COMMON TANGENT, LOOKING AT A PREVIOUS DIAGRAM (PART a)

$$m_{\text{(TANGENT)}} = +\frac{3}{4}$$

FINALLY WE HAVE, USING P(0, 5)

$$y - y_0 = m(x - x_0)$$

OR SIMPLY

$$\Rightarrow y = mx + c$$

$$\Rightarrow y = \frac{3}{4}x + 5$$

$$\Rightarrow 4y = 3x + 20$$

$$\Rightarrow 0 = 3x - 4y + 20$$

AS REQUIRED



# LYGB - MPI PAPER 0 - QUESTION 10

a) STARTING BY MANIPULATING THE FORMULA

$$\Rightarrow y = ab^x$$

$$\Rightarrow \log_{10} y = \log_{10}(ab^x)$$

$$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} b^x$$

$$\Rightarrow \log_{10} y = \log_{10} a + x \log_{10} b$$

$$\Rightarrow \log_{10} y = (\log_{10} b)x + \log_{10} a$$

↑  
Y

↑  
m

↑  
X

↑  
c

PREPARE THE DATA TO BE PLOTTED

$x = X$	5	10	15	20	25	30
$y$	1.7	4.5	11.0	26.0	70.0	160.0
$Y = \log_{10} y$	0.23	0.65	1.04	1.41	1.85	2.20

# IVGB - MPI PAPER 0 - QUESTION 10

## PLOTTING THE DATA



AS THE POINTS FORM A STRAIGHT LINE THE RELATIONSHIP IS INDEED OF THE FORM  $y = ab^x$

b) NOW WE HAVE BY COMPARING/READING VALUES

- $\log_{10} a = c$

$$\log_{10} a = -0.16$$

$$a = 10^{-0.16}$$

$$\underline{a \approx 0.7}$$

- $\log_{10} b = m$

$$\log_{10} b = \frac{1.8 - 0.24}{25 - 5}$$

$$\log_{10} b = 0.078$$

$$\underline{b \approx 1.2}$$

c) USING  $y = ab^x = 0.7 \times 1.2^x$  WITH  $x = 60$  WE OBTAIN  $\approx 39000$

## 1YGB - MPI PAGE 0 - QUESTION 11

BY PYTHAGORAS ON THE TRIANGLE ON THE "LEFT"

$$\Rightarrow a^2 + b^2 = c^2$$

$$\Rightarrow a^2 + b^2 - c^2 = 0$$

BY PYTHAGORAS ON THE TRIANGLE ON THE "RIGHT"

$$\Rightarrow (a+1)^2 + (b+1)^2 = (c+1)^2$$

$$\Rightarrow a^2 + 2a + 1 + b^2 + 2b + 1 = c^2 + 2c + 1$$

$$\Rightarrow (a^2 + b^2 - c^2) + 2a + 2b + 1 = 2c$$

$$\Rightarrow 0 + 2(a+b) + 1 = 2c$$

$$\Rightarrow 2(a+b) + 1 = 2c$$

L.H.S. WILL BE ODD IF  $a$  &  $b$  ARE BOTH INTEGERS

R.H.S. WILL BE EVEN IF  $c$  IS AN INTEGER

THENCE NOT ALL OF  $a, b$  &  $c$  ARE INTEGERS



# 1YGB - MPI PAPER 0 - QUESTION 12

a) START BY EXPANDING & COMPARING

$$f(x) \equiv (x-A)^3 - 4 \equiv x^3 - 6x^2 + 12x + B$$

$$\Rightarrow (x-A)(x-A)^2 - 4 \equiv x^3 - 6x^2 + 12x + B$$

$$\Rightarrow (x-A)(x^2 - 2Ax + A^2) - 4 \equiv x^3 - 6x^2 + 12x + B$$

$$\Rightarrow \left. \begin{array}{l} x^3 - 2Ax^2 + A^2x \\ - Ax^2 + 2A^2x - A^3 - 4 \end{array} \right) \equiv x^3 - 6x^2 + 12x + B$$

$$\Rightarrow x^3 - 3Ax^2 + 2A^2x - (A^3 + 4) \equiv x^3 - 6x^2 + 12x + B$$

LOOKING AT THE COEFFICIENTS OF  $x^2$  (x DOES NOT QUITE "WORK")

$$[x^2] - 3A = -6$$

$$A = 2$$

$$[x] 2A^2 = 12$$

$$A^2 = 4$$

NO UNIQUE  
ANSWER

$$[x^0] - A^3 - 4 = B$$

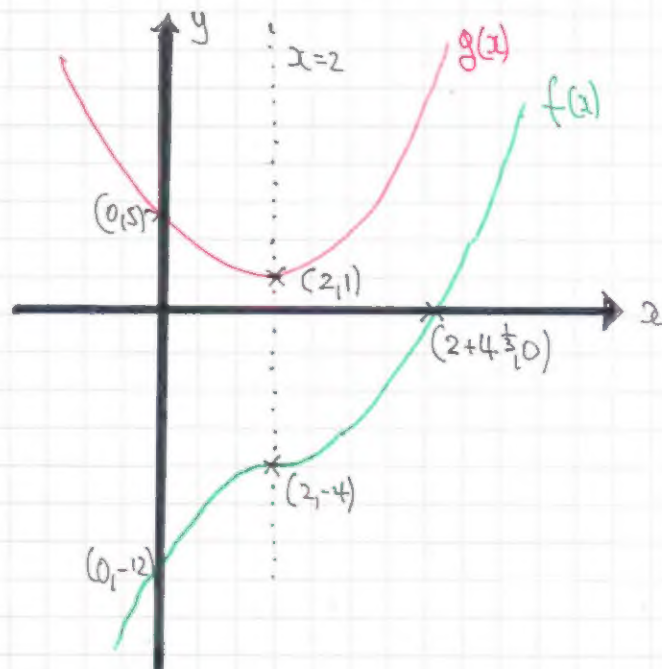
$$-8 - 4 = B$$

$$B = -12$$

b) •  $g(x) = x^2 - 4x + 5 = (x-2)^2 - 2^2 + 5 = (x-2)^2 + 1$

•  $f(x) = (x-2)^3 - 4$

$$\begin{aligned} f(0) &= (-2)^3 - 4 = -12 \\ f(x) &= 0 \\ \Rightarrow (x-2)^3 - 4 &= 0 \\ \Rightarrow (x-2)^3 &= 4 \\ \Rightarrow x-2 &= 4^{\frac{1}{3}} \\ \Rightarrow x &= 2 + 4^{\frac{1}{3}} \\ g(0) &= 5 \end{aligned}$$



1Y0B - MPI Paper 0 - Question 12

$$c) \Rightarrow x^3 - 7x^2 + 16x + B = .5$$

$$\Rightarrow x^3 - 7x^2 + 16x - 12 = .5$$

$$\Rightarrow x^3 - 6x^2 + 12x - 12 = x^2 - 4x + 5$$

$$\Rightarrow f(x) = g(x)$$

ALTHOUGH IT APPEARS FROM THE SKETCH THAT  
THE TWO GRAPHS DO NOT MEET THE CUBIC  
WILL EVENTUALLY "OVERTAKE" THE QUADRATIC FOR  
SUFFICIENTLY LARGE  $x$ .

$\therefore$  ONLY ONE ROOT //